# Combining Modal Concepts: Philosophical Applications*

Alexandre Costa-Leite

## 1. Introduction

Some mathematicians and computer scientists have the tendency to believe that modal logic is just about relational structures (i.e. structures composed by a set and relations on this set. Check for instance (Blackburn & De Rijke &Venema, 2001)). This is just one possible way to understand modal logic and, therefore, it does not imply that modal logic can be reduced to such a conception. Conceiving modal logics as “a tool for working with relational structures” (Blackburn et al., 2001) allows logicians, especially mathematically-oriented logicians, to unify a lot of different objects under the same label. However, for philosophical applications such a definition is not entirely adequate because it is not able to capture single philosophical aspects of concepts. Modal logic cannot be reduced to the study of Kripke semantics; nor can it be reduced to the research on what modalities such as possibility and necessity are. Indeed, one can find many definitions of modal logic in the literature. Some of the most important modal logicians have a lot of different conceptions of modal logic. At the very beginning of Hughes and Cresswell (1996), one finds the following remarks:

> “Modal logic is the logic of necessity and possibility, of ‘must be’ and ‘may be’. By this is meant that it considers not only truth and falsity applied to what is or is not so as things actually stand, but considers what would be so if things were different. If we think of how things are as the actual world then we may think of how things might have been as how things are in an alternative, non-actual but possible, state of affairs – or possible world.”

The above conception is clearly not founded in the “relational structures slogan”, but in a much more passionate account of modal logic. Such a conception can make someone think therefore that modal logic is not about the real world, but just about fictional worlds, because modal logicians are talking

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about *possible worlds* or *possible situations* and such entities are not the real world, although the real world is also a possible world. Nobody knows exactly what possible worlds are or if they really exist. Questions about the ontological status of possible worlds have been studied in the literature for a long time. David Lewis (1986) is one of the most famous philosophers who argues that possible worlds have an existence in the same way that the real world has. Such a conception generates an interesting philosophical discussion. Accepting the actual world as a constant realization of possible worlds (or possible worlds becoming real by updating reality), follows that some possible worlds, those which become real, have an ontological status and then really exist, given that they are the actual world.

Other interesting definition of modal logic is defended by Chagrov and Zakharyaschev (1997):

> "Modal logic is a branch of mathematical logic studying mathematical models of correct reasoning which involves various kinds of necessity-like and possibility-like operators."

It seems that both definitions of modal logic were targets of criticism specially developed by those people working on the "relational structures slogan". Blackburn et al. (2001) state the following:

> "One still encounters with annoying frequency the view that modal logic amounts to rather simple-minded uses of two operators $\Diamond$ and $\Box$. The view has been out of date at least since the late 1960's."

Such a comment attempts to establish a new approach to modal logic. Even if the "relational structures" are so fundamental to modal logic, there is no guarantee that in the future a new understanding of modalities will not change the way actual researchers on modal logic think about their subject.

Modal logic is interesting for philosophers because it is related to the metaphysical status of objects and with the content of an agent's mental states. In this sense, modal logic is the study of different existential dimensions of objects and the relations between such objects. For example, in the case of propositional logics, the objects to be considered are propositions and its different existential dimensions are expressed by modalities. Given a hierarchy of possibility operators, each one would be responsible for a given existential dimension of a given proposition. Modal logic, therefore, is a form of research that is concerned with the different ontological dimensions of objects and shows how to manipulate such dimensions.

In this article, such different dimensions of objects are considered in order to show how to apply combined modal logics in philosophy. Modal logic is helpful because it is a tool to clarify the analysis of philosophical concepts. Combining logics plays an important role in philosophical issues because there are some statements containing non-interdefinable operators which cannot be formalized using a single modal logic (i.e a modal logic with just one modal operator). The philosopher usually constructs and finds complicated propositions containing at the same time different modal notions. One example is that of the verification principle, which can be stated as: "All true propositions can be known" this principle often appears in discussions about realism and anti-realism. In the verification principle, one can find two non-interdefinable modalities: possibility and knowledge. Therefore, a very simple modal logic is not able to formalize such a sentence. (A detailed study to this problem is proposed in Costa-Leite, 2006.) Another example, the one treated in this article, is that of non-skepticism about the world. The statement "All contingent propositions are known" involves two non-interdefinable modalities: contingency and knowledge. It is difficult to find works studying in detail how to combine contingency and knowledge. Therefore, attempts to study non-skepticism fail while formalizing the statement. In this article, the philosophical statement above is studied from the viewpoint of combined logical systems. Indeed, one very simple method called fusion is applied to provide an example of formalization. The sense in which such complex formalisms can help in the understanding and formalization of statements linking metaphysics and epistemology will be explained.

## 2. Formal tools and philosophical concepts

The general theory of modalities still awaits some basic developments, considering that up to now it is not clear what modalities are and just what properties do modalities possess. There are a lot of different modalities and each modality is a particular way to modify a proposition updating its dimensional content. Given a proposition $\varphi$, one can always introduce to it a modality. One could create, for instance, $\Diamond\varphi$ (the possibility of $\varphi$) or $K\varphi$ (the knowledge of $\varphi$). Such modalities allow the construction of expressions of the form "$\varphi$ is possible" and "$\varphi$ is known", for example. The intuition and the study of modalities is important to understand other dimensions and properties of the actual world. Although introducing modalities in a given proposition allows statements of the above form, nothing can be said, from the viewpoint of non-combined modal systems, when multiple modalities are interacting in a proposition. In this paper, the interactions between two different families of modalities, those called *metaphysical* (or alethic) modalities and those called *epistemic* modalities are examined. While studying metaphysical and epistemic modalities there is also an attempt to provide explanations in metaphysics and epistemology, respectively. The study of formal concepts can help in the analysis and understanding of philosophical areas. However, it is important to note how such formal tools and concepts are limited. Van Benthem (2005) argued that:

> “Here is the worst that can happen. Some atlases of philosophical logic even copy philosophical geography (epistemic logic, deontic logic, alethic modal logic), leading to a bad copy of a bad map of reality”.

This statement seems to contain the key to discovering what is the exact role of formal methods in philosophy. What Van Benthem is arguing for is that formal tools can help, but cannot give an entire understanding of philosophical areas. And there is no doubt that sometimes a formal approach to some philosophical notion can even be a caricature of how to proceed. Van Benthem’s claim is correct. It is not reasonable to think that a formal study of metaphysical concepts will examine entirely, or even replace a realistic and intuitive approach, because many aspects of concepts cannot be formalized inside logical systems. In this sense, it is a mistake to think that a formal approach to a given concept can give a complete account to the whole of a given philosophical area. It seems that there will always be some controversy or problem. His statement is interesting to show in what sense reality and language are ingredients of two different things. Consider the formal and logical modality of possibility. Does this modality correspond to what possibility really is? It is hard to say. Formal tools help in the clarification and partial description of what a concept really is, but it is never able to describe the totality of the concept. One interesting property of formal concepts and tools is that some philosophical revolutions can be reached by a formal approach to concepts. One good example is that of Kripke (1980) who showed that there are some necessary *a posteriori* truths. Such a result shows exactly the right role of logic in philosophy: from one side, logic cannot eliminate all philosophical problems and it cannot give a total description of a given concept. But from the other side, the use of logic really helps to create some models of reality.

## 3. The problem

Gabbay (1999) pointed out the existence of a very interesting logical problem which reflects directly in philosophical issues. This is called the *fibring problem*. It can be explained in the following way: take a Kripke model $\langle W,R,v \rangle$ for $\Diamond$, a formula $\varphi$ and the complex modality $\Diamond K$. Given $\varphi$, introduce to it the combined modality in order to obtain $\Diamond K\varphi$. Now, to determine the truth-condition of the formula in the Kripke model one has to proceed in the standard way. However, when the truth-condition of the modality $K\varphi$ is examined, the above Kripke model cannot continue the procedure, because it is not able to recognize what K means. In this sense, Gabbay proposed to associate to each world a new model using something called the fibring function in order to be able to analyze the truth-condition of the complete formula. The idea of fibring is that sometimes the models are not sufficiently rich to determine truth-conditions of all propositions. Some many new variations of fibring have been proposed by many researchers in the branch called *combining logics.* A general approach to combined modalities and a great variety of references can be found in Costa-Leite (2004).

The fibring problem appears everywhere in philosophical analysis. In Costa-Leite (2006) tools from combining logics played an important role in studying in detail the exact set to formulate and think about a paradox related to the verification principle. In this work, a new example is provided using combinations of a metaphysical modality and an epistemic modality.

Metaphysical (or alethic) modalities are those related to the general structure of reality. The name *metaphysical* reflects this content. A metaphysical modality is one which is not directly related to the actual world, but with some potential configuration of this world. The name *alethic* suggests that the notion of truth appears in these modalities. The name alethic therefore is not a good one, because one can think that just alethic modalities deal with the notion of truth, what is incorrect. The general name *metaphysical* describes the job: modalities which state potential configurations of reality.

Epistemic modalities are not directly related to the general structure of reality, but rather with the cognitive status that an agent can have with respect to the world. The name *epistemic* suggests, of course, some relation between agents and the world. Epistemic modalities are also related to the concept of truth, especially when it comes to analyzing truth-conditions of epistemic formulas.

The study of metaphysical, deontic, epistemic, temporal and others kinds of modalities has been the target of much research. What has not been studied are conditions where one can find *interactions* of different families of modalities, as in the example above where the combination $\Diamond K\varphi$ appears. Some other examples of interactions are the following: $K\Diamond\varphi$, $K\varphi \rightarrow \bigtriangledown\varphi$ (knowledge implies contingency), $K\varphi \rightarrow \Diamond\varphi$ etc. There are a lot of cases. Such statements show propositions where two different families of modalities are interacting in a combined complex statement. The study of interactions between metaphysical and epistemic modalities deserves attention, because it provides a key to the door linking metaphysics and epistemology, and allows therefore a study of philosophical statements involving such concepts. Dana Scott (as cited in Hendricks & Symons 2006) correctly said:

> "Here is what I consider one of the biggest mistakes of all in modal logic: concentration on a system with just one modal operator. The only way to have any philosophically significant results in deontic logic or epistemic logic is to combine these operators with: Tense operators (otherwise how

can you formulate principles of change?); the logical operators (otherwise how can you compare the relative with the absolute?); the operators like historical or physical necessity (otherwise how can you relate the agent to his environment?); and so on and so on. (Dana Scott, 1970)"

### *3.1 The example*

One of the first examples, which is not explained here in detail, can be found in Costa-Leite (2006). There is showed that the right language and logic in which to formulate Fitch's paradox is composed by a fusion of modal languages and modal logics. In this sense, one can add the verification principle $\varphi \rightarrow \Diamond K\varphi$ to such a fusion without the collapse of truth and knowledge. The reader is invited to check that article to see how Fitch's paradox can be studied from the viewpoint of combined logics. Fusion of modal logics is a very simple method to combine modal logics. Such method has been studied mainly by Gabbay, but it has been discovered by Kracht & Wolter (1991), and also by Fine & Schurz (1997). The method is briefly explained in the example.

Consider the statement

(ST) "All contingent propositions are known."

There are many possible formalizations of the above sentence, it depends in what logic it is being formalized. First it is clear that a modal logic with just one modal operator cannot do the job. If one has just a metaphysical modal logic, then it is not able to formalize the knowledge operator. In the same way, with just an epistemic logic, it would not be possible to formalize contingency. Therefore, just a combined formalism can realize the task. But what is such combined modal logic?

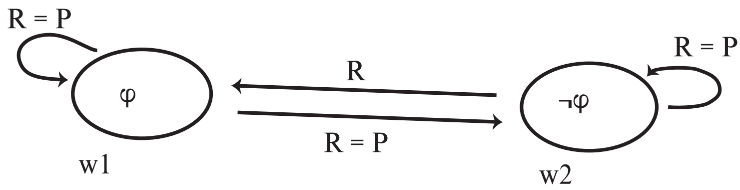
Logics of contingency were proposed by Montgomery & Routley (1966), and the authors presented a lot of systems taking contingency as a primitive operator. Such an approach is followed here (i.e. contingency as a primitive modality). Surely, if contingency is not taken as primitive, but defined using possibility, then the formalization is different. The contingency of a formula $\varphi$ is represented by $\triangledown\varphi$. One can read such formula as "$\varphi$ is contingent". Contingency of a formula $\varphi$ means that $\varphi$ is possible and it is possible non-$\varphi$. Epistemic logics use K to formalize knowledge. Consider now a language containing $<\&,\rightarrow,v,\sim,\triangledown>$ and a language containing $<\&,\rightarrow,v,\sim,K>$. A language containing $\triangledown$ and K among its symbols is certainly a logic able to formalize

(ST') "If a proposition is contingent, then it is known."

The language <&,→,v,~,▽,K> is called a fusion of the above structures. (Check Gabbay (1999) for details on fusions of modal logics.)

(ST) and (ST') are equivalent ways to announce the non-skeptical thesis. Such a thesis intends to show that the world is an object of knowledge. The first conclusion of this paper is that to formalize the non-skeptical thesis one needs a fusion of two languages, one for contingency and the other for knowledge.[1] But what is the logic of such a language? What does it semantics looks like?

The answers to the above problems depend of what the reader intend to do, assuming that there is no absolute answer. Using the fused language, the formalization of (ST) or (ST') is: $\nabla\varphi \rightarrow K\varphi$. The fused axiomatic system generated using such a language determines whether (ST) is valid or not (the same for (ST')). Such axioms certainly contain at least the axioms of a minimal modal logic of contingency and minimal epistemic logic. Fusion of two axiomatic systems A1 and A2 consists in putting together both axiomatic systems in a big set which contains all axioms of both A1 and A2, and all inference rules of both (check Gabbay (1999) for a detailed study on fusions). Surely, from the semantical viewpoint, fusion of two Kripke structures F1 and F2 consists in putting together both accessibility relations. In this sense, if F1 = <W,R> is a structure for contingency, and F2 = <W,P> is a structure for knowledge, the fusion of both is the structure F1⊕F2 = <W,R,P>. The accessibility relations of the fused structure have the same properties of the original accessibility relations. It means for instance that if R is reflexive in F1, then R is also reflexive in the fusion. Let consider here a fusion where the accessibility relation R is reflexive and symmetric, but P is just reflexive. Consider semantically (ST). Assume a fused Kripke model F1⊕F2 = <W,R,P,v>, the formula $\nabla\varphi \rightarrow K\varphi$ and put $P \subseteq R$, such that both are reflexive and symmetric.Take W = {w1,w2} and the following valuation: v(φ) = {w1}. In such a model, the formula is not valid, and therefore it is not a theorem of the fused logic, given that completeness is preserved by fusions (check Gabbay (1999) for details on completeness preservation). See the picture below:




[1] Classical propositional language can be viewed as a fusion of a language containing just negation and a language containing, for instance, conjunction. It is important to state that it is a fusion because it is now clear what method is used to generate such a language.

In the world w1 and w2, $\nabla\varphi$ holds, but in w1 and w2 $K\varphi$ does not hold. In this sense, the formula is not valid in the model. Surely one could create a modal logic showing that (ST) is valid, but again it depends of what one intends to do. What is important to state is that a complex formula involving two non-interdefinable modalities cannot be analyzed from the semantical viewpoint without a combined modal system able to understand at the same time what each one of the modalities means.

## 4. Conclusion

Combining modal concepts allows the study of complex statements formulated in natural languages. Such kind of approach provides a formal study on many different philosophical statements. In the example studied in this text, a concept from metaphysics (contingency) is linked to a concept from epistemology (knowledge) using a fusion of Kripke models. In this sense, combining concepts formally generates new insights in the study of the bridges between many different philosophical subjects.[2]

University of Neuchâtel
Switzerland
alexandre.costa-leite@unine.ch
http://luna.unine.ch/alexandre.costa/costaleite.html

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