The combined logics of skepticism

Alexandre Costa Leite
The City University of New York
Swiss National Science Foundation

Introduction

- In this work, we study relations between metaphysical and epistemic concepts;
- In particular, we are interested in interconnections between knowledge and contingency, and belief and contingency;
- In this sense, we are able to formalize a special form of skepticism called here *logical skepticism*.

Logical skepticism I

The logical skepticism defends that:

1) The world is contingent. Propositions about the world are contingent;
2) Propositions about the world are not known (and cannot be known);
3) But if a proposition is about the world, then one believes (or not) in such proposition;

Logical skepticism II

The motivations of the logical skeptic are:

1) Assuming that the world changes its configuration, and propositions which are true could be false, it follows that propositions about the world are contingent;
2) The problem of induction.

Logical skepticism III

The following are propositions containing some formulations of different versions of logical skepticism:

1) If a proposition is contingent, then it is not known;
2) If a proposition is contingent, the it cannot be known;
3) If a proposition is contingent, then it is not known, and its negation is not known;
4) If a proposition is contingent, then it cannot be known, and its negation cannot be known.

Formalizing logical skepticism I

- The framework required to formalize logical skepticism is that proposed by fusions of modal logics extended with interaction axioms;
- Consider a language containing the contingency operator $L_{\nabla} = < \neg, \wedge, \vee, \rightarrow, \nabla >$

  and a language containing the knowledge operator $L_K = < \neg, \wedge, \vee, \rightarrow, K >$

Formalizing logical skepticism II

- Thus, a fusion of both languages is a structure containing both operators:

$$L_{\nabla} \oplus L_K = < \neg, \wedge, \vee, \rightarrow, \nabla, K >$$

In such a language, propositions containing at the same « knowledge » and « contingency » can be generated. Therefore, such a fused structure is able to formalize the some versions of logical skepticism.

Formalizing logical skepticism III

$$(\boldsymbol{\nabla}\phi \longrightarrow \neg \boldsymbol{K}\phi)$$

$$(\boldsymbol{K}\phi \longrightarrow \boldsymbol{\Delta}\phi)$$

$$(\boldsymbol{\nabla}\phi \longrightarrow (\neg \boldsymbol{K}\phi \wedge \neg \boldsymbol{K}\neg\phi))$$

$$(\boldsymbol{\nabla}\phi \longrightarrow (\neg\Diamond \boldsymbol{K}\phi \wedge \neg\Diamond \boldsymbol{K}\neg\phi))$$

Fusions of modal logics and interaction axioms I

- Given that there are interactions and operators of different families in the formulations of logical skepticism, it follows that we should combine two different formalisms in order to understand skepticism.
- In this sense, we have to combine contingency logics with epistemic logics.
- Our candidates to be combined are reflexive modal logics;

Fusions of contingency and epistemic axiomatic systems

- Given one axiomatic system for knowledge, and a given axiomatic system for contingency, then their fusion consists in a big system containing both systems:

1. All tautologies of classical propositional logic CPL;
2. $(K(\phi \to \psi) \wedge K\phi) \to K\psi$;
3. $K\phi \to \phi$;
4. $\Box\phi \leftrightarrow \neg(\phi \to \nabla\phi)$
5. $\nabla\phi \leftrightarrow \nabla\neg\phi$
6. $\phi \to (\neg\nabla(\phi \to \psi) \to (\nabla\psi \to \nabla\phi))$
7. $\vdash \phi$ implies $\vdash K\phi$
8. $\vdash \phi$ then $\vdash \neg\nabla\phi$

Fusions of Kripke structures: contingency and knowledge

- Given a Kripke structure for knowledge and a Kripke structure for contingency, their fusion is a complex frame:

$F_K \oplus F_{T_\Delta} = < W, R, S >$ such that:

1. W is a non-empty set of possible worlds;
2. R is an accessibility relation for contingency: $R \subseteq W \times W$
3. S is an accessibility relation for knowledge: $S \subseteq W \times W$

Interactions

- Given a fusion of logics (languages, axiomatic systems and Kripke structures), the following automatic interaction appears:

$$(K\phi \to (\neg\nabla(\phi \to \psi) \to (\nabla\psi \to \nabla\phi)))$$

- Very interesting fact, given that fusions in general do not generate interactions

Fusions of modal logics and interaction axioms II

In general, when we fuse axiomatic systems, there is no interaction between axioms;

In this sense, we should expand the fusions with interaction axioms representing different versions of logical skepticism

Fusions of modal logics and interaction axioms III

- In this sense, we should add the above versions of the skeptical principle to the selected fusion. In this case, the fusion of T for knowledge and T for contingency.

$$T^* \oplus T_{\nabla} \oplus (\nabla\phi \to \neg K\phi)$$

$$T^* \oplus T_{\Delta} \oplus (K\phi \to \Delta\phi)$$

$$T^* \oplus T_{\nabla} \oplus (\nabla\phi \to (\neg K\phi \wedge \neg K\neg\phi))$$

$$T^* \oplus T_{\nabla} \oplus (\nabla\phi \to (\neg\Diamond K\phi \wedge \neg\Diamond K\neg\phi))$$

Fusions again

- Fusions are operations realized in three different levels: languages, axiomatic systems (proof-theoretical viewpoint) and Kripke frames (semantical viewpoint). The model for those logics are combined models containing two different accessibility relations, one for knowledge and the other for contingency, both are reflexive.

Completeness and interactions

"Our transfer results make it unnecessary to establish completeness and other properties separately for stratified multimodal logics, as long as these properties are known to hold for their monomodal components. Thus the results have applications in all areas in which several modal operators which do not interact logically are used." (K. FINE and G. SCHURZ in [41])

"The formation of fusions , or independent joins , is the simplest and perhaps most frequently used way of combining logics. Let $\boldsymbol{L_1}$ and $\boldsymbol{L_2}$ be two multimodal logics formulated in languages L_1 and L_2 , both containing the language L of classical propositional logic, but having disjoint sets of modal operators. Denote by $L_1 \oplus L_2$ the union of L_1 and L_2 . Then the fusion $\boldsymbol{L_1} \oplus \boldsymbol{L_2}$ of $\boldsymbol{L_1}$ and $\boldsymbol{L_2}$ is the smallest multimodal logic $\boldsymbol{L}$ in the language $L_1 \oplus L_2$ containing $L_1 \cup L_2$. In particular, if $\boldsymbol{L_1}$ is axiomatized by a set of axioms AX1 and $\boldsymbol{L_2}$ is axiomatized by AX2 , then $\boldsymbol{L_1} \oplus \boldsymbol{L_2}$ is axiomatized by the union $AX1 \cup AX2$. This means that no axiom containing modal operators from both languages L_1 and L_2 is required to axiomatize the fusion of $\boldsymbol{L_1}$ and $\boldsymbol{L_2}$. The modal operators in $\boldsymbol{L_1}$ and $\boldsymbol{L_2}$ remain *independent*, they do not interact..." (GABBAY ET AL in [44])

Completeness preservation

- For pure fusions (without interactions), completeness is usually preserved: if the L1 is complete, and the logic L2 is complete, then their fusion is also complete (Kracht, Wolter, Fine and Schurz);
- But we do not have any criteria to state that completeness is preserved when we also add those interactions.

Contingency and belief

- The same methodology applies also for combinations between contingency and belief. The logical skeptic, although (s)he defends that the world cannot (and it is not) known, it is (can be) object of belief.
- In this sense, the logical skeptic could be formalized in the following way (assuming the fused language containing B and nabla):

Contingency and belief

$$\nabla\phi \rightarrow B\phi$$

$$\nabla\phi \rightarrow \Diamond B\phi$$

$$\nabla\phi \rightarrow (B\phi \vee B\neg\phi)$$

$$\nabla\phi \rightarrow (\Diamond B\phi \vee \Diamond B\neg\phi)$$

The combined logic of skepticism

- As one can see, there are many possible formulations of logical skepticism, one of the most interesting (full logical skepticism) is the following one (generated in a triple fusion of languages, axioms and frames):

$$T^* \oplus T_{\nabla} \oplus KD45 \, (\nabla\phi \rightarrow (\neg K\phi \wedge \neg K\neg\phi)) \oplus (\nabla\phi \rightarrow (\Diamond B\phi \vee \Diamond B\neg\phi))$$

$$(\nabla\phi \rightarrow (\neg\Diamond K\phi \wedge \neg\Diamond K\neg\phi)) \oplus (\nabla\phi \rightarrow (\Diamond B\phi \vee \Diamond B\neg\phi))$$

The combined logic of skepticism

- The fused language has three operators, the fused axiomatic systems contains three axiomatic systems, and the models contain three accessibility relations;
- A logical skeptic interested in defending logical skepticism should just show that his/her system is complete.

The combined logic of skepticism

- If such a completeness proof can be reached, then the logical skeptic has good logical grounds to state his position against dogmatics;
- And at the same time, good arguments (induction, contingency of the world), and a strong conception of knowledge to announce his position.