Imagination and the Square of Opposition

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Outline

- A Previous Approach to the Problem.

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- The Squares that Arise from those Essays.

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- Our Ideas.

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- His approach follows the methods of Hintikka's formulations of logics of propositional attitudes.
- This means that an agent a imagines a proposition φ, written $I_a\varphi$, and the operator I_a is modeled by some 'normal-like' modal logic.
- The logic can only be 'normal-like' for reasons similar to those of epistemic logic.

- Niiniluoto's logic validates the two axioms:
  - $I_a(\varphi \supset \psi) \supset (I_a\varphi \supset I_a\psi)$, and
  - $I_a(\varphi \wedge \psi) \equiv (I_a\varphi \wedge I_a\psi)$.

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  - $I_a(\varphi \wedge \psi) \equiv (I_a\varphi \wedge I_a\psi)$.
- But if it validated $I_a\tau$, where τ represents any tautology that would be very counterintuitive. So the logic is not normal even though the two axioms above hold.

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- How one might construct, or what a $< I, a >$-compatible would look like is not expounded in detail.

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- We think this is a crucial failure of Niinluoto's account. Even though he does make explicit that he is not looking for a system of rules that one's mind ought to follow, he is looking for a *semantics* for the term 'imagines'.
- But there is a problem with the claims he makes with his axioms, the particulars of which we will deal with below.
- What about the phenomenological aspect of imagination?

- If there are cases where Niinluoto’s axioms fail then there is a failure to be a theory for this kind of attitude.

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- Thus, we must take into account possible phenomenological counterexamples.
- In fact Niinluoto does look to justify some of his axioms, including those that deal with possibility and imagination.
- Now we can consider some squares of imagination.

The Squares of Imagination

According to Niinluoto, along with the axioms above, it is not possible to imagine a logical contradiction. Thus the axiom

$$I_a\varphi \supset \Diamond\varphi$$

is validated. With these we get the following squares

$$
\begin{array}{ccc}
I_a\varphi & \text{———} & I_a\neg\varphi \\
\downarrow & & \downarrow \\
\neg I_a\neg\varphi & \text{———} & \neg I_a\varphi
\end{array}
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I_a\varphi & \text{———} & \neg\Diamond\varphi \\
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$$

We have two squares, one of which is a “pure" square of imagination, and one that displays an interaction between imagination and possibility. In fact these two are related because the fact that imagination implies possibility then implies, given the other axioms of the system, that imagining φ implies that one doesn’t imagine $\neg\varphi$.

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- We do not think that imagining one thing excludes imagining its negation, as it might usually be understood.

- Indeed, we think that imagining that it is raining, and imagining that it is not raining is possible.

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- one might liken this to watching two movies on two screens at the same time. Watching one does not mean that we are not watching the other.
- Note that this is different then imagining the contradiction.

- Now, rejecting that idea that we cannot imagine something and its negation separately does not mean that we must give up the axiom $I_a\varphi \supset \Diamond\varphi$.

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- However, what we do give up are the other two axioms, or at least the adjunctive axiom.
- As we can all agree we can imagine physical impossibilities, but what about the logically impossible?

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- What exactly do we mean by 'logic' when we say logically impossible?
- There are other logics than classical logic to use, and if we consider quantum logic we can then, in a sense, imagine a situation that is classically impossible.
- Is that imagining the impossible then?

- We must broaden our concept of imagination to include the kind of imagining that one does in mathematics.

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- Consider what it is to imagine a set, a picture of a set is not a set because that is something like a spacial object which sets are not.
- Consider, then, a mathematical theorem. If mathematical truths are necessary truths, if not logical truths, then when we consider a theorem true that is not we have imagined an impossibility.

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- not everything in a story is imagined as pictured, but these things may have consequences for a story.
- For instance, that the characters are standing on a solid floor is imagined, but the floor is left unimagined. Although that part of the story is important for the story's coherence.

- Thus, something is imagined in this sense that its implications are imagined. The consequences are the important aspect of a theorem and it is how we judge its truth or falsity.

- Thus, something is imagined in this sense that its implications are imagined. The consequences are the important aspect of a theorem and it is how we judge its truth or falsity.
- It is not that a set of all sets is incoherent in itself, but that it implies incoherency that we judge it as wrong.

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- Following that rejection what can we offer as squares of imagination.
- Lloyd Humberstone has offered a method of analyzing modal notions through squares thus it makes sense to try his suggestion.

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- Then we can characterize the relationship between two operators, $\oplus_1, \oplus_2$ as contrary or subcontrary in the following way.
- Contrary: $\oplus_1 \wedge \oplus_2 = \bot$, $\oplus_1 \vee \oplus_2 \neq \top$, $\oplus_1 = \oplus_2\neg$ and $\oplus_1\neg = \oplus_2$.

- Or the conjunction of the operators is always the constant false operator, the disjunction is not always the constant true operator. Finally, the first is the post negation of the other.

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- So the conjunction is not the false operator, the disjunction is the true operator and they are the post-negation of one another.

- So the first question to ask is should the imagination operator take the A or E corner? We choose to treat imagination as a kind of possibility so it takes the E corner.

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- Now the post negation of I_a is to say imagine not φ. It is true that, under our assumptions, $I_a\varphi$ and $I_a\neg\varphi$ can both be true.

- So the first question to ask is should the imagination operator take the A or E corner? We choose to treat imagination as a kind of possibility so it takes the E corner.
- Now the post negation of I_a is to say imagine not φ. It is true that, under our assumptions, $I_a\varphi$ and $I_a\neg\varphi$ can both be true.
- However, one of $I_a\varphi$ or $I_a\neg\varphi$ does not have to be true. Unless we say something like imagining a dog wearing a sweater is sufficient for imagining that my best friend is not in jail. But that seems wrong.

- Thus, Humberstone’s method doesn’t seem to bare fruit. So we can reject the idea that imagination can be treated as a modal operator, or we can find another way to approach the problem.

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- What we can find are instances of subalternations. Consider the idempotency axioms.
- $I_a I_a \varphi \supset I_a \varphi$ and $I_a \varphi \supset I_a I_a \varphi$

- Neither of these seem like they will work. One can imagine that they imagine that φ with out imagining φ itself. And if one imagines φ, then they don't have to imagine that one is imagining that φ.

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- What we are tempted to conclude is that there is no 'Pure Square' for imagination.
- So we must consider imagination mixed with another operator.

Imagination and Possibility

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- There are really two options $\Diamond I_a$ and $I_a\Diamond$.
- The first is what we might say is a translation of ‘imaginable’ the second is then to say that we see something as a possibility.

- First dealing with imaginable, we say that if it is imagined then it is imaginable. Thus, $I_a\varphi \supset \Diamond I_a\varphi$ holds. But the converse does not.

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- There are also things that are not imaginable. Thus, just because it is possible does not mean that it is imaginable. So we don't validate $\Diamond\varphi \supset \Diamond I_a\varphi$.

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- There are also things that are not imaginable. Thus, just because it is possible does not mean that it is imaginable. So we don't validate $\Diamond\varphi \supset \Diamond I_a\varphi$.
- With the next axiom we may seem a bit normative rather than descriptive.

- If something is imagined then we see what is imagined as a possibility, or that it is possible.

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- So $I_a\varphi \supset I_a\Diamond\varphi$ is an axiom.
- One may suggest that this must be wrong if we assert that one can imagine impossibilities.
- But we think this is precisely what we do. We make errors in thinking that what we imagine is indeed possible. I think that this is something that Kant was worried about.

- But the converse is not the case. Imagining that φ is possible, is not always to imagine φ.

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- Imagining the possibility of seeing infrared light and imagining seeing infrared light are two different things.
- We then can summarize these into the following squares.

$$
\begin{array}{ccc}
I_a\varphi & \text{———} & \neg\Diamond I_a\varphi \\
\downarrow & & \downarrow \\
\Diamond I_a\varphi & \text{———} & \neg I_a\varphi
\end{array}
$$

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\begin{array}{ccc}
I_a\varphi & \text{———} & \neg I_a\Diamond\varphi \\
\downarrow & & \downarrow \\
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- What we are in need of is something to fill out the following square:

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$$\begin{array}{ccc} ? & \text{———} & \neg I_a\varphi \\ \downarrow & & \downarrow \\ I_a\varphi & \text{———} & ? \end{array}$$

- The problem that we face here is that if one does not imagine φ, then nothing is a necessary consequence of that.

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- So it does not seem likely that we can find something to fit in these empty corners.