Combining knowledge and contingency

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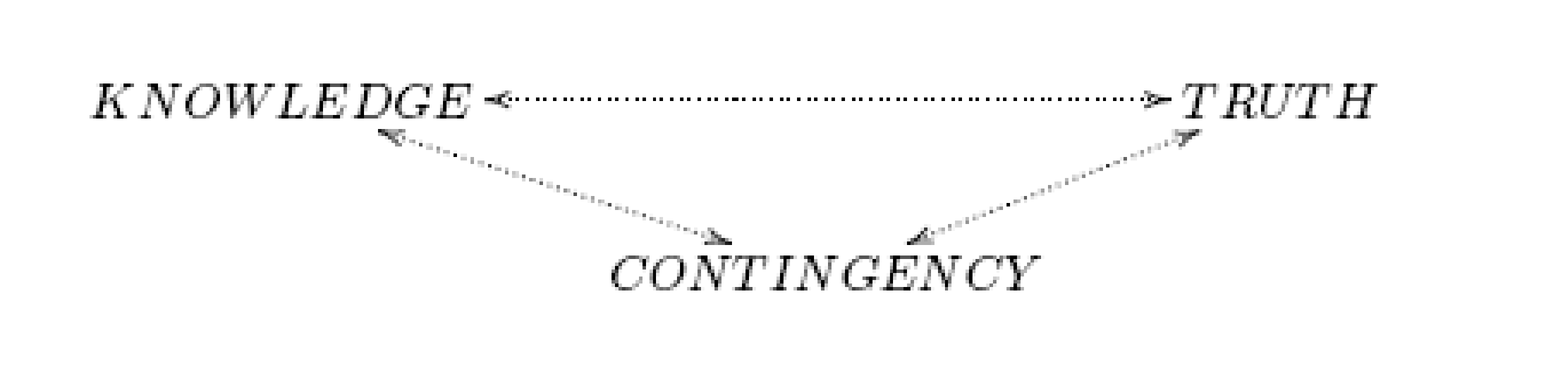
Introduction

Costa-Leite, A. (2007). Interactions of metaphysical and epistemic concepts. Université de Neuchâtel, PhD Thesis.

- Interactions of metaphysical and epistemic concepts: contingency and knowledge - bridges between metaphysics and epistemology.
- Contingency logics and its philosophical consequences;
- Epistemic logics;
- Combining contingency logics and epistemic logics: fusions of modal logics;
- Logical skepticism

Examples of interactions: the case of non-interdefinable concepts

- If a proposition is contingent, then it is known;
- If a proposition is known, then it is absolute;
- If a proposition is true, then it can be known;
- If a proposition can be known, then it is not contingent.

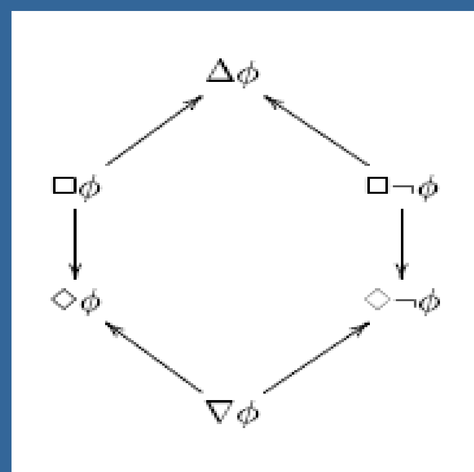


What is contingency?

- Metaproperty contingency: tautologies, contradictions and contingencies;
- Modal contingency: « p » is contingent if and only if p is possible and the negation of p is also possible; $\nabla \phi = \Diamond \phi \wedge \Diamond \neg \phi$
- Absolute: « p » is absolute if and only if p is necessary or its negation is necessary;

Contingency and the Square of Opposition

- Robert Blanché proposed the following configuration of Aristotle's square:



BLANCHÉ, R. (1969). *Structures Intellectuelles: essai sur l'organisation systématique des concepts.* Paris, Librairie Philosophique J. Vrin.

Contingency logics

- Proposed by MONTGOMERY, H. ROUTLEY. (1966). Contingency and non-contingency bases for normal modal logics. *Logique et Analyse*, vol.9.
- Given a modal language containing the contingency operator, it follows that:

The system T^{∇} is defined by:

1. $\Box\phi \leftrightarrow \neg(\phi \rightarrow \nabla\phi)$
2. $\nabla\phi \leftrightarrow \nabla\neg\phi$
3. $\phi \rightarrow (\neg\nabla(\phi \rightarrow \psi) \rightarrow (\nabla\psi \rightarrow \nabla\phi))$

   And by the rule:

4. $\vdash \phi$ then $\vdash \neg\nabla\phi$

Non-contingency logics

Defining non-contingency; Non-contingency logics are composed by the following:

1. $\Delta\phi \leftrightarrow \Delta\neg\phi$
2. $\phi \rightarrow (\Delta(\phi \rightarrow \psi) \rightarrow (\Delta\phi \rightarrow \Delta\psi))$

And the definition and rule below:

1. $\Box\phi =^{def} \phi \wedge \Delta\phi$
2. $\vdash \phi$ then $\vdash \Delta\phi$

Epistemic logics

Knowledge: truth in all possible worlds; knowledge as necessity

- Usual systems of epistemic logic
- Epistemic logic and its applications...

Mixing knowledge and contingency: fusions of languages – fusions of modal logic (Gabbay, Wolter, Kracht, Fine, Schurz)

- Given a language containing contingency , and given a language containing knowledge, their fusion is a language containing both operators;
- In such a language, interactions of contingency and knowledge can be formalized and studied in detail;

Fusions of contingency and epistemic axiomatic systems

- Given one axiomatic system for knowledge, and a given axiomatic system for contingency, then their fusion consists in a big system containing both systems:

1. All tautologies of classical propositional logic CPL;
2. $(K(\phi \to \psi) \wedge K\phi) \to K\psi$;
3. $K\phi \to \phi$;
4. $\Box\phi \leftrightarrow \neg(\phi \to \nabla\phi)$
5. $\nabla\phi \leftrightarrow \nabla\neg\phi$
6. $\phi \to (\neg\nabla(\phi \to \psi) \to (\nabla\psi \to \nabla\phi))$
7. $\vdash \phi$ implies $\vdash K\phi$
8. $\vdash \phi$ then $\vdash \neg\nabla\phi$

Fusions of Kripke structures: contingency and knowledge

- Given a Kripke structure for knowledge and a Kripke structure for contingency, their fusion is a complex frame:

$F_K \oplus F_{T_\Delta} = \langle W, R, S \rangle$ such that:

1. W is a non-empty set of possible worlds;
2. R is an accessibility relation for contingency: $R \subseteq W \times W$
3. S is an accessibility relation for knowledge: $S \subseteq W \times W$

Interactions

- Given a fusion of logics (languages, axiomatic systems and Kripke structures), the following automatic interaction appears:

$$(K\phi \rightarrow (\neg\nabla(\phi \rightarrow \psi) \rightarrow (\nabla\psi \rightarrow \nabla\phi)))$$

- This is a very interesting fact, given that fusions in general do not generate interactions, although its language can be used to formalize interactions

Logical skepticism

The logical skeptic argues that:

1) The world is contingent. Propositions about the world are contingent. Therefore, the world is not known, assuming that there is just knowledge of necessary truths;

2) The problem of induction. Therefore, the world is not known.

Formalizing logical skepticism

- Logical skepticism « The world is not known » can be reduced to the following statement: If a proposition is contingent, then it is not known (involving therefore an interaction of knowledge and contingency);
- The framework of fusions is perfect for modelling such interaction.

Preserving completeness

- Given two complete logics, their fusion preserves completeness (Kracht & Wolter (1991) and Fine & Schurz (1996)
- The system of contingency logic is complete, and the system of epistemic logic is also complete. Thus, their fusion is complete

1. $T^* \oplus T_{\nabla}$
2. $T^* \oplus T_{\Delta}$

Logical skepticism

- Given that a proposition is known, it follows that it is not contingent. The premise states that « p is known ». But knowledge implies truth. By the rule

  $\vdash \phi$ then $\vdash \neg\nabla\phi$ it follows that it is not contingent.

The logical skeptic should show that the metatheorem of deduction holds in the fusion.

What should a logic skeptic do?

Another way: a logical skeptic interested in proving the truth of its position should show that:

1. $T^{*} \oplus T_{\nabla} \oplus (\nabla\phi \rightarrow \neg K\phi)$

2. $T^{*} \oplus T_{\Delta} \oplus (K\phi \rightarrow \Delta\phi)$

is a complete logic.

Logical skepticism

- If he/she is able to prove the completeness of such a system, then logical skepticism is an acceptable theory from the logical viewpoint.

Is the logic of skepticism complete?